SUBJECT: DECISION SCIENCE

Module 3: Network Analysis

Comprehensive University-Level Study Notes • Units 12 – 15

12 Network Analysis – Introduction, Rules for constructing a Network, Different Time Calculations

Introduction to Network Analysis

Network Analysis is a powerful operations management technique used to plan, schedule, monitor, and control complex, large-scale projects consisting of multiple interrelated dependencies. A project is broken down into distinct, identifiable items known as activities and milestones. Network analysis visually models these variables as a graphical network structure, allowing project managers to detect bottlenecks, optimize resource allocation, and predict structural project completion timelines accurately.

Core Components of a Network Diagram

- **Activity:** A specific task or operation within the project lifecycle that consumes resources, labor, or time. Represented visually as an arrow (→).
  - *Critical Activity:* An activity with zero float; any delay immediately shifts the total project deadline.
  - *Dummy Activity:* A structural dependency link that consumes zero time and zero resources. Represented as a dashed arrow, it is introduced solely to preserve logical relationships between distinct nodes.
- **Event (Node):** A specific point in time representing the start or completion of one or more activities. Nodes do not consume time or resources. Represented visually as a circle.
  - *Merge Event:* A node where multiple incoming activities intersect.
  - *Burst Event:* A node where multiple outgoing activities originate.

Rules for Constructing a Network Diagram

To avoid logical distortions, network diagrams must strictly satisfy the following rules:

1. Every activity must be represented by exactly one arrow. No single activity can be represented multiple times in a network.
2. Arrows must flow logically from left to right, indicating the forward progression of time.
3. **No Looping (Cyclical Paths):** A network loop occurs when a sequence of activities circles back to a previously completed node. This is invalid as it creates an endless timeline loop.
4. **No Dangling:** An activity cannot be left disconnected in space. Every activity must connect back to a final project completion node unless it is the terminal event.
5. **Fulkerson's Numbering Rule:** Nodes must be numbered systematically. An initial event node is selected and assigned the number 1. All outgoing arrows originating from node 1 are deleted to find next-stage baseline source nodes, which are numbered sequentially (2, 3, 4...). This ensures the tail event number is always strictly smaller than the head event number ($i < j$).

Different Time Calculations

Project scheduling uses a two-pass computational framework (Forward Pass and Backward Pass) to isolate critical milestones:

I. The Forward Pass (Earliest Time Metrics)

Calculates the earliest possible time an event can occur or an activity can start. It moves from left to right across the network diagram, utilizing the **Maximum Method** at merge events:

> *Earliest Start Time (EST) of activity (i, j) = Earliest Occurrence Time of node i (E_i)*
> *Earliest Finish Time (EFT) of activity (i, j) = EST + t_{ij} (Where t_{ij} is activity duration)*
> *$E_j = Max [E_i + t_{ij}]$ (Calculated across all incoming activities at node j)*

II. The Backward Pass (Latest Time Metrics)

Calculates the latest possible time an event can occur without delaying the total project completion timeline. It moves from right to left, utilizing the **Minimum Method** at burst events:

> *Latest Finish Time (LFT) of activity (i, j) = Latest Occurrence Time of node j (L_j)*
>
> *Latest Start Time (LST) of activity (i, j) = LFT − t_{ij}*
>
> *L_i = Min [$L_j - t_{ij}$] (Calculated across all outgoing activities from node i)*

Understanding Float and Slack Metrics

Float refers to the amount of time an activity can be delayed without affecting the project completion timeline. There are four types of floats used in operations audits:

- **Total Float (TF):** The total amount of time an activity can be delayed without altering the final project deadline.
  Formula: TF = LFT – EFT or TF = LST – EST or TF = $L_j - E_i - t_{ij}$
- **Free Float (FF):** The portion of float that allows an activity to be delayed without affecting the Earliest Start Time of any subsequent dependent activities.
  Formula: FF = $E_j - E_i - t_{ij}$ or FF = Total Float – Head Slack (S_j)
- **Independent Float (IF):** The safest float measure; the delay allowed if the preceding activities end as late as possible and succeeding activities start as early as possible.
  Formula: IF = $E_j - L_i - t_{ij}$ or IF = Free Float – Tail Slack (S_i) (If negative, it is treated as 0).
- **Interfering Float (IntF):** The measure of float variation that causes a direct reduction in the float values of downstream activities.
  Formula: IntF = Total Float – Free Float or IntF = Head Slack (S_j)

13 CPM and PERT - Time Estimation, Critical Path

Time Estimation Frameworks

Network analysis maps operational timelines through two distinct paradigms depending on environmental predictability: the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT).

I. CPM Time Estimation (Deterministic Approach)

CPM assumes high historical predictability (e.g., standard civil construction, routine maintenance). It relies on a single, fixed, deterministic time estimate per activity. No probability metrics are used, as resource costs and durations are assumed known with certainty.

II. PERT Time Estimation (Probabilistic Approach)

PERT is designed for unique, non-repetitive, high-uncertainty projects (e.g., advanced research and development pipelines, software rollouts). It handles timeline uncertainty by gathering **three distinct time estimates** per activity, modeled across a Beta Probability Distribution curve:

1. **Optimistic Time (t_o):** The absolute minimum time required if everything goes perfectly under ideal operational conditions.
2. **Most Likely Time (t_m):** The normal, expected duration under typical operating conditions; the mode of the probability distribution.
3. **Pessimistic Time (t_p):** The maximum time required if significant bottlenecks, delays, or structural breakdowns occur (excluding major natural acts).

Using these variables, decision scientists calculate the statistical Mean and Variance for each activity:

$$\text{Expected Mean Time } (t_e) = (t_o + 4t_m + t_p) / 6$$

$$\text{Variance } (\sigma^2) = [(t_p - t_o) / 6]^2$$

$$\text{Standard Deviation } (\sigma) = (t_p - t_o) / 6$$

The Critical Path Engine

The **Critical Path** is the longest continuous sequence of dependent activities running through a network diagram from project initiation to final terminal node completion. This path determines the minimum possible duration required to complete the total project. Any delay to an activity sitting on this path causes a matching delay to the final delivery date.

Algorithmic Identification of Critical Paths

To identify the absolute critical path across a network matrix, locate nodes and activities that satisfy three simultaneous mathematical identities:

1. $E_i = L_i$ (Earliest and Latest times match at the tail event node)
2. $E_j = L_j$ (Earliest and Latest times match at the head event node)
3. $E_j - E_i = t_{ij}$ or $L_j - L_i = t_{ij}$ (The duration between nodes exactly equals activity time)

Activities satisfying these criteria have a **Total Float of Zero**. Summing up the durations along this zero-float path yields the total expected project duration.

14 Merits and Demerits of CPM & PERT

Implementing network analysis models provides substantial optimization benefits while introducing unique operational challenges:

Merits of CPM and PERT

- **Structured Project Visualisation:** Forces project managers to detail all activities and interdependencies before project execution, reducing coordination oversights.
- **Precise Bottleneck Identification:** Isolates the critical path, informing management exactly which activities cannot afford delays. This prevents wasted administrative oversight on non-critical paths.
- **Optimized Resource Allocations:** Highlights high-float (non-critical) activities, allowing managers to safely reallocate idle machinery, labor, or capital to support critical activities.
- **Empirical Risk Analysis:** PERT enables probabilistic project modeling, allowing managers to calculate the exact percentage probability of meeting a specific corporate deadline using standard Z-score normal distribution values.

Demerits of CPM and PERT

- **Garbage-In, Garbage-Out Risk:** The mathematical reliability of the network model depends heavily on the accuracy of the underlying time estimates. Subjective or biased duration inputs distort the entire analytical output.
- **High Administrative Overhead:** Maintaining a large network diagram with hundreds of interdependent lines requires continuous data logging, updates, and specialized software tracking systems.

- **Rigid Dependency Assumptions:** Network structures assume clear sequences. In highly creative or rapidly shifting environments, task sequences change dynamically, making fixed network models obsolete quickly.
- **Over-Focus on the Critical Path:** Managers can focus so heavily on tracking critical activities that they ignore near-critical paths. A near-critical activity with a small float can easily become critical if ignored, causing unexpected project delays.

15 Difference between PERT and CPM

While both tools utilize network graph architectures to manage project timelines, they originate from different engineering paradigms and are deployed across fundamentally distinct corporate scenarios.

Comparison Basis	PERT Framework	CPM Framework
Core Philosophy	Probabilistic in nature; handles high operational uncertainty.	Deterministic in nature; assumes fixed operational constants.
Operational Focus	Event-oriented; prioritizes hitting milestones.	Activity-oriented; prioritizes managing individual task execution.
Time Estimates	Utilizes 3 time metrics (Optimistic, Most Likely, Pessimistic).	Utilizes a single, deterministic time estimate based on history.
Optimal Application	Non-repetitive, unique assignments (R&D, space exploration, software development).	Repetitive, routine commercial setups (civil construction, plant shutdowns, shipping audits).
Probability Curve	Models timelines across a Beta Probability Distribution.	No probability distribution utilized.
Cost Optimization	Does not natively handle cost calculations; focuses heavily on time control.	Features built-in **Time-Cost Tradeoff (Crashing)** algorithms.
Critical Path Dummy	Rarely uses dummies; relies on event sequencing structures.	Frequently uses dummy activities to clarify parallel task limits.

Strategic Integration Concept

In modern industrial applications, the distinction between PERT and CPM has blurred. Advanced project management information systems routinely merge the probabilistic uncertainty calculations of PERT with the structured time-cost crashing algorithms of CPM, providing unified decision-making engines for enterprise optimization.

End of Module 3 • Subject: Decision Science