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Bachelor of Business Administration (BBA) | Fifth Semester | UK5DSCMGT303

Management Science

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Module 5: Project Management and Network Analysis

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Chapter 1: Project Management Fundamentals and Network Modeling Principles

1.1 Meaning, Characteristics, and the Project Triple Constraint

In contemporary business administration and operations research, a **Project** is defined by the Project Management Institute (PMI) as a temporary endeavor undertaken to create a unique product, service, or operational outcome. Projects are distinguished from routine recurring operations by five defining characteristics:

- **Uniqueness:** Every project involves a non-repetitive set of activities (e.g., constructing a metro rail transit corridor, launching an enterprise ERP software platform, or designing a supersonic commercial airliner).
- **Definite Lifespan:** A project has a clearly defined beginning date and an unambiguous scheduled completion milestone date.
- **Interconnected Sub-tasks:** A project consists of dozens or hundreds of discrete, interdependent activities bound by strict precedence relationships.
- **Cross-Functional Resource Deployment:** Demands the coordinated integration of specialized engineering personnel, financial capital, physical machinery, and outsourced subcontractors.
- **The Triple Constraint (Iron Triangle):** Project managers must balance three competing dimensions: **Scope & Quality**, **Time (Schedule)**, and **Cost (Budget)**. Compressing project duration inevitably inflates direct expenditures or risks quality compromises.

1.2 Network Diagram Topologies: AOA versus AON

To analyze, schedule, and control complex projects, operations researchers model project workflows using directed graph networks. Two topological paradigms exist:

Dimension	Activity-on-Arrow (AOA) Networks	Activity-on-Node (AON) Networks
Representation of Activities	Activities are represented by directed arrows (→); the length of the arrow does not reflect duration.	Activities are represented by rectangular or circular nodes; arrows depict only precedence relationships.
Representation of Milestones	Events (instants in time representing start or finish) are represented by circular nodes.	Milestones are represented by specialized zero-duration milestone nodes.
Need for Dummy Activities	Mandatorily requires dotted dummy arrows (– – – →) with zero duration and zero cost to preserve logic and prevent duplicate identity.	Completely eliminates dummy activities! All finish-to-start precedence is depicted by direct arrows between nodes.
Software Precedence	Traditional manual CPM/PERT formulation standard in academic textbooks.	Standard global convention in modern digital software (e.g., Microsoft Project, Primavera P6).

1.3 Network Construction Rules and Topologic Defects

When constructing Activity-on-Arrow (AOA) project networks, planners must adhere to strict mathematical conventions:

1. **Flow Orientation:** Networks flow chronologically from left to right. Arrows must point forward in time; retrograde arrows are prohibited.
2. **Unique Event Identification:** Each activity must be uniquely identified by its tail (starting) event i and head (finishing) event j such that $i < j$. No two activities can share identical starting and ending events.
3. **Fulkerson's Rule for Event Numbering:** Numbering starts at the initial project event with no incoming arrows (Event 1). Delete all outgoing arrows from Event 1; scan for new events with no incoming arrows and number them sequentially (2, 3, ...). Continue until the terminal event is numbered.
4. **Elimination of Topologic Errors:**
   - *Looping / Cycling:* An erroneous closed circular sequence of arrows where an activity depends upon its own subsequent completion.
   - *Dangling:* An activity ending at an isolated event without connecting to the final project completion event, falsely suggesting project termination.
   - *Redundant Dummy Activities:* Inserting unnecessary dummy arrows where existing physical precedence relationships already capture dependency logic.

Chapter 2: Critical Path Method (CPM) for Deterministic Projects

2.1 Forward Pass and Backward Pass Computations

The Critical Path Method (CPM) was developed in 1957 by J. E. Kelley of Remington Rand and M. R. Walker of DuPont to schedule industrial chemical plant overhauls. In CPM, activity durations are assumed to be **deterministic (known with certainty)**.

To determine project duration and activity schedules, planners execute a two-phase computational sweep across the network:

Phase 1: Forward Pass (Earliest Times Calculation)

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Begins at the initial project event at time zero ($ES_1 = 0$) and moves chronologically forward through the network:

- **Earliest Start Time (ES_{ij}):** The earliest possible point in time an activity can commence. Equal to the earliest time of its tail event: $ESij = Ei$.
- **Earliest Finish Time (EF_{ij}):** The earliest possible point in time an activity can finish:

```
EFij = ESij + tij
```

Legend: EF_{ij} = Earliest finish time; ES_{ij} = Earliest start time; t_{ij} = Duration of activity (i, j).

- **Earliest Event Time for Merge Events:** When multiple activities merge into an event j, that event cannot occur until **all** incoming activities have finished:

```
Ej = max(i)[EFij] = max(i)[Ei + tij]
```

Legend: E_j = Earliest occurrence time of event j; max taken over all activities terminating at event j.

Phase 2: Backward Pass (Latest Times Calculation)

Begins at the terminal project completion event N, setting its latest allowable time equal to its earliest finish time ($LN = EN$), and traces backward to time zero:

- **Latest Finish Time (LF_{ij}):** The latest point in time an activity can finish without delaying the overall project completion date. Equal to the latest time of its head event: $LFij = Lj$.
- **Latest Start Time (LS_{ij}):** The latest point in time an activity can commence without delaying the overall project:

```
LSij = LFij − tij
```

Legend: LS_{ij} = Latest start time; LF_{ij} = Latest finish time; t_{ij} = Duration of activity (i, j).

- **Latest Event Time for Burst Events:** When an event i bursts into multiple outgoing activities, its latest allowable time is bounded by the most restrictive outgoing activity:

```
Li = min(j)[LSij] = min(j)[Lj − tij]
```

Legend: L_i = Latest allowable time of event i; min taken over all activities originating from event i.

2.2 Critical Path Identification and Float Analysis

The **Critical Path** is the continuous sequence of connected activities extending from the initial event to the terminal event that possesses the longest aggregate duration. The duration of the critical path determines the **minimum total time required to complete the entire project**. Any delay in any critical activity causes an identical delay in the final project completion date!

Non-critical activities possess scheduling flexibility known as **Float (Slack)**:

Float Metric	Mathematical Formula	Managerial Interpretation & Utility
Total Float (TF)	$TFij = LSij − ESij = LFij − EFij = Lj − Ei − tij$	The maximum amount of time an activity can be delayed without delaying the final project completion date. Delaying an activity beyond its total float makes it critical and delays the project.
Free Float (FF)	$FFij = Ej − EFij = Ej − Ei − tij$	The amount of time an activity can be delayed without delaying the earliest start of any immediately succeeding activity. Float that belongs exclusively to that activity.
Independent Float (IF)	$IFij = \max[0, Ej − Li − tij]$	The amount of time an activity can be delayed when its tail event occurs at its latest allowable time and head event occurs at earliest time. Does not affect any preceding or succeeding activities.
Interfering Float (ITF)	$ITFij = TFij − FFij = Lj − Ej$	The portion of total float that, if consumed, delays the earliest start of succeeding activities without delaying final project completion. Exactly equals head event slack.

Chapter 3: Programme Evaluation and Review Technique (PERT)

3.1 Probabilistic Modeling and the Beta Distribution

The Programme Evaluation and Review Technique (PERT) was formulated in 1958 by the United States Navy Special Projects Office, alongside consulting firm Booz Allen Hamilton and Lockheed Corporation, to accelerate the development of the Polaris ballistic missile submarine program. Unlike CPM's deterministic assumptions, PERT is specifically designed for **non-deterministic, research-intensive, novel projects** where activity durations cannot be predicted with certainty.

To capture subjective engineering uncertainty, PERT employs a continuous **Beta Probability Distribution** parameterized by three distinct time estimates obtained from technical experts for each activity:

1. **Optimistic Time (a or to):** The shortest possible duration in which an activity can be executed if everything proceeds flawlessly under exceptionally favorable conditions. Probability of occurring is less than 1%.
2. **Most Likely Time (m or tm):** The modal duration that would occur most frequently if the activity were repeated multiple times under normal, typical operating conditions.
3. **Pessimistic Time (b or tp):** The longest duration required to complete the activity under adverse circumstances, accounting for severe delays, equipment breakdowns, and supplier failures (excluding catastrophic acts of God). Probability of occurring is less than 1%.

3.2 Statistical Formulas for Expected Duration and Variance

Under the standard PERT Beta distribution assumption, the modal value m receives four times the weight of extreme estimates:

Expected Activity Duration (te):

$$te = 6a+4m+b$$

Legend: t_e = Weighted mean expected activity duration; a = Optimistic time; m = Most likely time; b = Pessimistic time.

Activity Variance (σ2) and Standard Deviation (σ):

Assuming the spread between pessimistic and optimistic estimates represents six standard deviations (b – a = 6σ encompassing 99.7% of the distribution):

$$\sigma = 6b-a \implies \sigma 2 = (6b-a)2$$

Legend: sigma = Standard deviation of activity duration; sigma^2 = Variance of activity duration measuring uncertainty.

3.3 Project Variance and Completion Probability Calculation

By applying the **Central Limit Theorem**, when a project consists of a substantial number of independent activities along its critical path, the aggregate project completion duration (T) is approximately **Normally Distributed**:

- **Expected Project Duration (Te):** The sum of expected durations of all activities lying on the critical path:

  $$Te = \sum critical\ te$$

  Legend: T_e = Total expected project completion duration.

- **Total Project Variance (σP 2):** The sum of variances of all activities lying on the critical path (variances of non-critical activities are completely ignored):

  $$\sigma P\ 2 = \sum critical\ \sigma i2 \implies \sigma P = \sqrt{\sum critical\ \sigma i2}$$

  Legend: sigma_P^2 = Project variance; sigma_P = Standard deviation of overall project duration.

- **Standard Normal Deviate (Z):** To determine the probability that the project will be completed on or before a specified contract deadline date (Ts), calculate the standard normal variate:

  $$Z = \frac{Ts-Te}{\sigma P}$$

  Legend: Z = Number of standard deviations between target deadline T_s and expected duration T_e.

- The probability $P(T \le Ts) = \Phi(Z)$ is obtained directly from standard cumulative normal distribution tables.

Chapter 4: Project Crashing and Time-Cost Trade-Offs

4.1 Direct Costs, Indirect Costs, and Optimum Project Duration

In competitive project management, clients frequently offer contractual financial bonuses for accelerated completion or enforce severe liquidated damages for project delays. Project managers must evaluate **Time-Cost Trade-Offs**:

- **Direct Project Costs:** Expenditures directly attributable to individual activities—overtime wages, multi-shift operations, hiring extra equipment, or expedited freight. **Direct costs increase exponentially as project duration is compressed**.
- **Indirect Project Costs:** Ongoing managerial overheads, site supervision salaries, equipment rental leases, and project insurance. **Indirect costs decrease linearly as project duration is shortened**.
- **Total Project Cost Curve:** The sum of Direct and Indirect costs forms a U-shaped convex curve. The lowest point on this curve identifies the **Optimum Project Duration** that minimizes total capital expenditure.

4.2 Normal versus Crash Parameters and Cost Slope Formula

Every activity possesses dual operational parameters:

- **Normal Time (Nt) & Normal Cost (Nc):** Standard activity duration and direct cost under routine operating conditions.
- **Crash Time (Ct) & Crash Cost (Cc):** The absolute minimum duration into which the activity can be compressed by mobilizing maximum resources, and the corresponding elevated direct cost.

Cost Slope Formula (S): The direct financial cost incurred to crash an activity by one unit of time (e.g., per day or per week):

> $S = \text{Normal Time} - \text{Crash Time} \quad \text{Crash Cost} - \text{Normal Cost} = Nt - Ct \quad Cc - Nc$

Legend: S = Incremental direct cost per day of schedule compression; C_c = Direct crash cost; N_c = Normal direct cost; N_t = Normal duration; C_t = Crash duration.

4.3 Systematic Step-by-Step Crashing Protocol

To compress a project schedule economically, execute the following protocol:

1. Calculate the Cost Slope (S) for every activity in the network.
2. Identify the Critical Path and compute current project duration.
3. Identify all activities lying on the critical path that can still be crashed (Nt > Ct).
4. Select the critical activity possessing the **absolute lowest Cost Slope (min S)**. Crashing a non-critical activity wastes money without reducing project duration!
5. Crash that activity by the maximum permissible amount until: (a) its crash limit is reached, or (b) a new parallel path becomes critical.
6. Update network times. If parallel critical paths emerge, subsequent crashing requires simultaneously compressing all parallel critical paths by picking combinations with the lowest combined cost slope.
7. Continue crashing until all critical paths reach their absolute crash limits or until the daily cost slope exceeds daily indirect cost savings.

Chapter 5: Comprehensive Worked Numerical Problems

5.1 Problem 1: CPM Network Analysis and Full Float Schedule

A civil construction project consists of eight distinct engineering activities—**A, B, C, D, E, F, G, H**. The precedence dependencies and normal durations (in weeks) are tabulated below:

Activity	Immediate Predecessors	AOA Events (i → j)	Normal Duration (Weeks)
A	None (Start activity)	1 → 2	4 weeks
B	None (Start activity)	1 → 3	6 weeks
C	Activity A	2 → 4	5 weeks
D	Activity B	3 → 4	3 weeks
E	Activity B	3 → 5	7 weeks
F	Activities C, D	4 → 6	6 weeks
G	Activity E	5 → 6	4 weeks
H	Activities F, G	6 → 7	3 weeks

Step 1: Forward Pass (Earliest Times)

- Event 1: $E_1 = 0$
- Event 2: $E_2 = E_1 + tA = 0 + 4 = 4$
- Event 3: $E_3 = E_1 + tB = 0 + 6 = 6$
- Event 4 (Merge): $E_4 = \max[E_2 + tC, E_3 + tD] = \max[4 + 5, 6 + 3] = \max[9, 9] = 9$
- Event 5: $E_5 = E_3 + tE = 6 + 7 = 13$

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- Event 6 (Merge): $E6 = \max[E4 + tF, E5 + tG] = \max[9 + 6, 13 + 4] = \max[15, 17] = 17$
- Event 7 (Terminal): $E7 = E6 + tH = 17 + 3 = 20$ Weeks

Total Minimum Project Completion Duration = **20 Weeks**.

Step 2: Backward Pass (Latest Times)

- Event 7: $L7 = E7 = 20$
- Event 6: $L6 = L7 - tH = 20 - 3 = 17$
- Event 5: $L5 = L6 - tG = 17 - 4 = 13$
- Event 4: $L4 = L6 - tF = 17 - 6 = 11$
- Event 3 (Burst): $L3 = \min[L4 - tD, L5 - tE] = \min[11 - 3, 13 - 7] = \min[8, 6] = 6$
- Event 2: $L2 = L4 - tC = 11 - 5 = 6$
- Event 1 (Burst): $L1 = \min[L2 - tA, L3 - tB] = \min[6 - 4, 6 - 6] = \min[2, 0] = 0$

Step 3: Comprehensive Float Schedule

Activity	Events	Time (t)	ES	EF	LS	LF	Total Float	Critical Status
A	1 → 2	4	0	4	2	6	2	Non-Critical
B	1 → 3	6	0	6	0	6	**0**	**CRITICAL**
C	2 → 4	5	4	9	6	11	2	Non-Critical
D	3 → 4	3	6	9	8	11	2	Non-Critical
E	3 → 5	7	6	13	6	13	**0**	**CRITICAL**
F	4 → 6	6	9	15	11	17	2	Non-Critical
G	5 → 6	4	13	17	13	17	**0**	**CRITICAL**
H	6 → 7	3	17	20	17	20	**0**	**CRITICAL**

Critical Path Determination

The unique Critical Path connects activities possessing zero total float: **B → E → G → H** (Events: 1 → 3 → 5 → 6 → 7).

Total Project Duration = 6 + 7 + 4 + 3 = 20 Weeks.

5.2 Problem 2: PERT Three Time Estimates and Probability Analysis

An aerospace engineering firm is designing a satellite sensor package. The three time estimates (in months) for the critical activities of the project are:

Critical Activity	Optimistic (a)	Most Likely (m)	Pessimistic (b)	Expected Duration: te
Activity 1-2	2 mos	5 mos	8 mos	(2 + 20 + 8)/6 = **5.0 mos**
Activity 2-4	3 mos	6 mos	15 mos	(3 + 24 + 15)/6 = **7.0 mos**
Activity 4-5	4 mos	7 mos	10 mos	(4 + 28 + 10)/6 = **7.0 mos**
Activity 5-6	2 mos	4 mos	6 mos	(2 + 16 + 6)/6 = **4.0 mos**
Total Critical Path	**Expected Project Duration (Te)**			**23.0 Months**

Step 1: Activity Variances along Critical Path: $\sigma 2 = [(b - a)/6]2$

- Activity 1-2: $\sigma 12 = [(8 - 2)/6]2 = [6/6]2 = (1.0)2 = 1.00$
- Activity 2-4: $\sigma 22 = [(15 - 3)/6]2 = [12/6]2 = (2.0)2 = 4.00$
- Activity 4-5: $\sigma 32 = [(10 - 4)/6]2 = [6/6]2 = (1.0)2 = 1.00$
- Activity 5-6: $\sigma 42 = [(6 - 2)/6]2 = [4/6]2 = (0.667)2 = 0.444$

Step 2: Total Project Variance and Standard Deviation

$$\sigma P\,2 = 1.00 + 4.00 + 1.00 + 0.444 = 6.444 \implies \sigma P = \sqrt{6.444}$$

$$= 2.538 \text{ Months}$$

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Legend: sigma_P = Standard deviation of the overall critical path duration.

Step 3: Probability Calculation for Client Contract Milestone (Ts = 26 Months)

> $Z = \sigma P\ Ts - Te = 2.53826.0 - 23.0 = 2.5383.0 = +1.18$

Legend: Z = Standard normal variate.
From cumulative standard normal distribution tables, the probability for Z = +1.18 is Φ(1.18) = 0.8810.

Managerial Probability Conclusion: There is an **88.10% probability** that the aerospace satellite project will be completed within the 26-month contractual deadline.

Chapter 6: Comprehensive Worked Numerical Problem in Project Crashing

6.1 Problem Statement and Activity Parameters

An industrial manufacturing facility overhaul project comprises six activities—**A, B, C, D, E, F**. The project indirect overhead costs are **Rs 400 per day**. The precedence relationships, normal durations, crash durations, normal costs, and crash costs are tabulated below:

Activity	Predecessor	Normal Time	Crash Time	Normal Cost	Crash Cost	Cost Slope (S)
A	None (Start)	4 days	3 days	Rs 1,000	Rs 1,400	Rs 400 / day
B	None (Start)	6 days	4 days	Rs 1,200	Rs 1,800	Rs 300 / day
C	Activity A	2 days	1 day	Rs 800	Rs 1,200	Rs 400 / day
D	Activity B	5 days	3 days	Rs 1,500	Rs 2,100	Rs 300 / day
E	Activity C	6 days	3 days	Rs 1,800	Rs 3,000	Rs 400 / day
F	Activity D	7 days	4 days	Rs 2,100	Rs 3,600	Rs 500 / day
Total	**Baseline Direct Normal Cost**			**Rs 8,400**	-	-

Cost Slope Calculation Verification: $S = Nt - CtCc - Nc$

- Activity A: (1,400 - 1,000) / (4 - 3) = 400 / 1 = **Rs 400 per day** (max crash = 1 day)
- Activity B: (1,800 - 1,200) / (6 - 4) = 600 / 2 = **Rs 300 per day** (max crash = 2 days)
- Activity C: (1,200 - 800) / (2 - 1) = 400 / 1 = **Rs 400 per day** (max crash = 1 day)
- Activity D: (2,100 - 1,500) / (5 - 3) = 600 / 2 = **Rs 300 per day** (max crash = 2 days)
- Activity E: (3,000 - 1,800) / (6 - 3) = 1,200 / 3 = **Rs 400 per day** (max crash = 3 days)
- Activity F: (3,600 - 2,100) / (7 - 4) = 1,500 / 3 = **Rs 500 per day** (max crash = 3 days)

6.2 Step-by-Step Network Crashing Iterations

There are two parallel paths connecting project start to project completion:

- **Path 1: A → C → E:** Normal duration = 4 + 2 + 6 = **12 days**.
- **Path 2: B → D → F:** Normal duration = 6 + 5 + 7 = **18 days** (Critical Path!).

Baseline Normal Project Duration = **18 days**.
Direct Cost = Rs 8,400. Indirect Cost = 18 days × Rs 400 = Rs 7,200. Total Cost = Rs 8,400 + Rs 7,200 = **Rs 15,600**.

Crashing Iteration 1 (Crash Path 2 by 2 days):

- Critical Path is Path 2 (B - D - F). Critical activities are B (slope 300), D (slope 300), F (slope 500).
- Lowest cost slope is Rs 300 per day (tie between Activity B and Activity D). Select Activity B.
- Crash Activity B by its full limit of **2 days** (duration reduces from 6 days down to 4 days).
- Path 2 duration becomes: 4 + 5 + 7 = **16 days**. Path 1 remains 12 days. Path 2 is still uniquely critical!
- Direct Cost increases by: 2 days × Rs 300 = Rs 600 → Direct Cost = 8,400 + 600 = Rs 9,000.
- Indirect Cost decreases to: 16 days × Rs 400 = Rs 6,400.
- Total Project Cost = Rs 9,000 + Rs 6,400 = **Rs 15,400** (Net savings = Rs 200!).

Crashing Iteration 2 (Crash Path 2 by 2 days):

- Next lowest cost slope on Path 2 is Activity D with slope Rs 300 per day.
- Crash Activity D by its full limit of **2 days** (duration reduces from 5 days down to 3 days).
- Path 2 duration becomes: 4 + 3 + 7 = **14 days**. Path 1 remains 12 days. Path 2 is still uniquely critical!

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- Direct Cost increases by: 2 days × Rs 300 = Rs 600 → Direct Cost = 9,000 + 600 = Rs 9,600.
- Indirect Cost decreases to: 14 days × Rs 400 = Rs 5,600.
- Total Project Cost = Rs 9,600 + Rs 5,600 = **Rs 15,200** (Net savings = Rs 200!).

Crashing Iteration 3 (Crash Path 2 by 2 days to meet Path 1 at 12 days):

- On Path 2, only Activity F can be crashed (slope Rs 500 per day).
- Notice: Crash cost (Rs 500) exceeds daily indirect savings (Rs 400)! Compressing schedule by 1 day will increase direct cost by 500 while saving only 400 in indirect overhead, raising total cost by Rs 100 per day.
- However, if management must achieve a client delivery milestone of 12 days, crash Activity F by 2 days (duration becomes 5 days).
- Project duration becomes **12 days** (both Path 1 and Path 2 are now simultaneously critical at 12 days).
- Direct Cost = 9,600 + (2 × 500) = Rs 10,600. Indirect Cost = 12 × 400 = Rs 4,800. Total Cost = **Rs 15,400**.

6.3 Comprehensive Crashing Optimization Summary

Crashing Stage	Project Duration	Direct Cost (Rs)	Indirect Cost (Rs)	Total Cost (Rs)
Stage 0 (Normal)	18 days	Rs 8,400	Rs 7,200	Rs 15,600
Stage 1 (Crash B)	16 days	Rs 9,000	Rs 6,400	Rs 15,400
Stage 2 (Crash D)	**14 days (OPTIMUM)**	**Rs 9,600**	**Rs 5,600**	**Rs 15,200 (MINIMUM)**
Stage 3 (Crash F)	12 days	Rs 10,600	Rs 4,800	Rs 15,400

Managerial Decision Rule: The optimal economic project duration is **14 days**, achieving the global minimum total project expenditure of **Rs 15,200**. Compressing beyond 14 days causes direct overtime premiums to outpace indirect cost reductions.

Chapter 7: Resource Allocation, Smoothing, and Leveling

7.1 Resource Histograms and Daily Demand Profiles

Traditional CPM and PERT models assume that organizational resources—such as mobile cranes, certified welders, or diagnostic software licenses—are available in unlimited supply. In commercial reality, daily resource availability is strictly capped by budget limits or physical equipment availability.

To analyze resource commitments over time, project managers compile a **Resource Loading Histogram**, plotting calendar project time on the horizontal axis and total units of resource consumed per day on the vertical axis:

- If all activities commence at their Earliest Start Times (ES), high resource demand spikes occur during early project phases, followed by idle valleys later.
- If activities commence at their Latest Start Times (LS), severe resource congestions are delayed until the project deadline, exposing the firm to catastrophic delays if any disruption occurs.

7.2 Resource Leveling versus Resource Smoothing

Operations researchers distinguish between two formal resource optimization techniques:

Analytical Dimension	Resource Smoothing	Resource Leveling (Allocation)
Primary Constraint	Project completion time is fixed and strictly inflexible.	Maximum daily resource availability ceiling is strictly fixed.
Mathematical Mechanism	Non-critical activities are shifted forward or backward within their available **Total Float** (TF).	Activities are delayed beyond available float, resulting in an extension of overall project duration.
Impact on Critical Path	Zero impact. Critical path and project duration remain completely unchanged.	Extends the critical path. Lengthens scheduled project completion milestone date.
Managerial Objective	Eliminate sharp peak-and-valley hiring/firing cycles; achieve smooth uniform workforce utilization.	Ensure that peak daily resource demand never violates physical equipment or union workforce caps.

Chapter 8: Comparative Architecture: CPM versus PERT

Comparison Dimension	Critical Path Method (CPM)	Programme Evaluation and Review (PERT)
Intellectual Origin	Developed by DuPont and Remington Rand for industrial chemical plant maintenance (1957).	Developed by US Navy, Booz Allen Hamilton, and Lockheed for Polaris missile defense (1958).

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Comparison Dimension	Critical Path Method (CPM)	Programme Evaluation and Review (PERT)
Nature of Activity Durations	**Deterministic:** Single time estimate based on historical empirical operating records.	**Probabilistic:** Three time estimates (optimistic, most likely, pessimistic) modeled via Beta distribution.
Orientation Paradigm	**Activity-Oriented:** Primary focus is on optimizing individual work tasks and cost slopes.	**Event-Oriented:** Primary focus is on monitoring milestone accomplishment and probability thresholds.
Cost & Crashing Focus	Directly integrates time-cost trade-offs and explicit crashing algorithms via Cost Slopes.	Primary focus is on time duration variance; cost considerations are handled separately via PERT/Cost.
Ideal Application Sectors	Repetitive civil construction, plant overhauls, building construction, recurring manufacturing.	Non-repetitive R&D projects, aerospace defense systems, prototype design, pharmaceutical clinical trials.

Chapter 9: Project Tracking via Earned Value Management (EVM)

9.1 Foundational Metrics of Earned Value Management

Modern enterprise project governance requires continuous monitoring of cost and schedule performance using standardized Earned Value Management (EVM) metrics:

- **Planned Value (PV / BCWS):** Budgeted Cost of Work Scheduled. The approved capital budget allocated to scheduled work planned to be completed up to the reporting date.
- **Earned Value (EV / BCWP):** Budgeted Cost of Work Performed. The approved budget value of work that has actually been physically completed up to the reporting date.
- **Actual Cost (AC / ACWP):** Actual Cost of Work Performed. The total realized financial expenditure incurred in executing the completed work.

9.2 Variance and Performance Indices Formulas

Performance variances measure absolute deviations from planned baselines:

$$\text{Schedule Variance: } SV = EV - PV \quad \bullet \quad \text{Cost Variance: } CV = EV - AC$$

Legend: SV > 0 indicates ahead of schedule; CV > 0 indicates under budget (favorable cost performance).

Performance indices measure efficiency ratios:

$$SPI = \frac{EV}{PV} \quad \text{and} \quad CPI = \frac{EV}{AC}$$

Legend: SPI = Schedule Performance Index (SPI >= 1.0 means ahead of schedule); CPI = Cost Performance Index (CPI >= 1.0 means spending under budget).

Forecasting Estimate at Completion (EAC): If current cost performance efficiency is expected to persist through remaining project phases:

$$EAC = \frac{BAC}{CPI}$$

Legend: EAC = Projected total project cost at completion; BAC = Budget at Completion (original total project budget).

Chapter 10: Comprehensive Worked Practical Problem in Earned Value Management

10.1 Project Diagnostic Scenario and Baseline Parameters

An enterprise information systems modernization project has an approved total budget at completion of **BAC = Rs 60,00,000** and an approved master schedule duration of **12 months**. At the end of **Month 6**, the project management office conducts a formal performance audit. The project records reveal the following accounting and technical status:

- **Planned Value (PV / BCWS):** According to the master schedule, 50% of total project work was scheduled to be completed by Month 6:

$$PV = 0.50 \times 60,00,000 = \text{Rs } 30,00,000$$

Legend: PV = Budgeted Cost of Work Scheduled up to Month 6.

- **Earned Value (EV / BCWP):** Forensic engineering audits confirm that exactly 40% of physical work packages have been completed and verified against quality standards:

$$EV = 0.40 \times 60,00,000 = \text{Rs } 24,00,000$$

Legend: EV = Budgeted Cost of Work Performed (value of work actually accomplished).

- **Actual Cost (AC / ACWP):** Financial general ledger accounts indicate that actual cash expenditures incurred to date total:

$$AC = Rs\ 28,00,000$$

Legend: AC = Actual Cost of Work Performed (realized money disbursed).

10.2 Step-by-Step EVM Diagnostic Computations

1. Schedule Variance (SV) and Cost Variance (CV):

$$SV = EV - PV = 24,00,000 - 30,00,000 = -Rs\ 6,00,000$$

Legend: Negative SV indicates the project is seriously behind schedule (Rs 6 Lakhs worth of planned work uncompleted).

$$CV = EV - AC = 24,00,000 - 28,00,000 = -Rs\ 4,00,000$$

Legend: Negative CV indicates the project is experiencing an unfavorable cost overrun (exceeded earned budget by Rs 4 Lakhs).

2. Schedule Performance Index (SPI) and Cost Performance Index (CPI):

$$SPI = PVEV = 30,00,00024,00,000 = 0.80$$

Legend: SPI of 0.80 means the project is progressing at only 80% of its planned schedule velocity.

$$CPI = ACEV = 28,00,00024,00,000 = 0.857$$

Legend: CPI of 0.857 means the enterprise receives only 85.7 paise of tangible project value for every Rupee spent!

3. Critical Ratio (CR):

$$CR = SPI \times CPI = 0.80 \times 0.857 = 0.686$$

Legend: Critical Ratio well below 1.0 (0.686) signifies severe compounded distress requiring urgent executive intervention.

4. Forecasting Final Project Cost at Completion (EAC):

Assuming that current cost burn inefficiencies will continue unabated for remaining work:

$$EAC = CPIBAC = 0.85760,00,000 = Rs\ 70,01,167$$

Legend: EAC = Projected total expenditure upon project completion.

$$VAC = BAC - EAC = 60,00,000 - 70,01,167 = -Rs\ 10,01,167$$

Legend: VAC = Variance at Completion forecasting an aggregate cost overrun of approximately Rs 10 Lakhs.

5. To-Complete Performance Index (TCPI):

To determine the cost efficiency the remaining project team must achieve to complete the project within the original budget ceiling of Rs 60,00,000:

$$TCPI = BAC-ACBAC-EV = 60,00,000-28,00,00060,00,000-24,00,000 = 32,00,00036,00,000 = 1.125$$

Legend: Remaining work must achieve a CPI of 1.125 (12.5% above original standard efficiency) to avoid overall budget breach.

Chapter 11: Work Breakdown Structure (WBS) and Cumulative S-Curves

11.1 Work Breakdown Structure Architecture

Effective project scheduling cannot occur without systematic decomposition of complex project deliverables into manageable operational work packages. The **Work Breakdown Structure (WBS)** is a hierarchical, deliverable-oriented tree structure that partitions the total project scope into five descending tiers:

WBS Level	Structural Entity	Operational Function & Corporate Governance
Level 1	Total Project Final Deliverable	Overarching contractual objective (e.g., Commercial Airport Terminal Construction).

WBS Level	Structural Entity	Operational Function & Corporate Governance
Level 2	Major Sub-Projects / Phases	Primary functional sub-systems (e.g., Civil Runway, Passenger Terminal Building, Air Traffic Tower).
Level 3	Sub-Deliverables / Systems	Specialized engineering systems (e.g., Baggage Handling System, HVAC Climate Control, Security Scanners).
Level 4	Work Packages (Cost Accounts)	The lowest manageable unit assigned to a specific control account manager with defined budget and timeline.
Level 5	Individual Network Activities	Discrete operational work tasks scheduled directly onto CPM/PERT network diagrams.

11.2 Cash Flow Forecasting and Cumulative S-Curves

When monthly project budgets are aggregated cumulatively over time, the resulting financial trajectory forms an **S-Curve**. The S-curve exhibits three characteristic phases:

- **Initial Acceleration Phase (Flat Start):** Expenditures are modest during early project phases as design studies, stakeholder clearances, and procurement orders are initiated.
- **Middle Execution Phase (Steep Rise):** Maximum capital burn rate occurs during full-scale fabrication, civil construction, and assembly, represented by the steep slope of the S-curve.
- **Final Commissioning Phase (Tapering Plateau):** Expenditures taper off as punch-list corrections, regulatory inspections, and client handover are completed.

Chapter 12: Advanced Network Construction and Topologic Diagnostics

12.1 Comprehensive 10-Activity Network Analysis Case

To synthesize all network modeling concepts, consider a high-technology industrial engineering project comprising ten distinct activities with complex finish-to-start dependencies:

Activity Code	Immediate Predecessors	AOA Network Path	Deterministic Duration (Days)
Act A	None (Start)	1 → 2	5 days
Act B	None (Start)	1 → 3	8 days
Act C	None (Start)	1 → 4	6 days
Act D	Activity A	2 → 5	7 days
Act E	Activity B	3 → 5	4 days
Act F	Activity B, C	3 → 6 (via Dummy 4→3)	9 days
Act G	Activity D, E	5 → 7	6 days
Act H	Activity F	6 → 7	5 days
Act I	Activity G	7 → 8	4 days
Act J	Activity H	7 → 8 (concurrent)	4 days

Critical Path Network Resolution

Evaluating the multiple parallel pathways through the network:

- Path 1: A → D → G → I: 5 + 7 + 6 + 4 = **22 days**.
- Path 2: B → E → G → I: 8 + 4 + 6 + 4 = **22 days**.
- Path 3: B → F → H → J: 8 + 9 + 5 + 4 = **26 days (CRITICAL PATH!)**.
- Path 4: C → Dummy → F → H → J: 6 + 0 + 9 + 5 + 4 = **24 days**.

Minimum Project Duration: Bounded by the longest continuous sequence of activities along Path 3: **26 Days**.

Total float on Path 1 = 26 - 22 = 4 days. Total float on Path 2 = 26 - 22 = 4 days. Total float on Path 4 = 26 - 24 = 2 days. The project manager must allocate top monitoring priority to Activities B, F, H, and J to prevent overall schedule delay.

Chapter 13: Project Risk Management and Monte Carlo Simulation

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13.1 Systematic Risk Governance Lifecycle

Modern mega-projects operate under extreme volatility, geopolitical uncertainty, supply chain fragility, and technological obsolescence. Project Risk Management is the structured discipline of identifying, analyzing, mitigating, and monitoring project risks throughout the project lifecycle. The PMBOK framework partitions risk management into four structured phases:

Phase	Key Analytical Methodologies	Deliverables & Strategic Controls
1. Risk Identification	Delphi expert consensus panels, structured brainstorming, prompt lists, Ishikawa Root Cause fishbone diagrams, checklist analysis.	Comprehensive **Project Risk Register** cataloging risk descriptions, potential trigger events, and designated risk owners.
2. Qualitative Risk Analysis	Probability and Impact (P-I) assessment mapping risks on a 5x5 severity matrix. Calculating composite Risk Score: $R = P \times I$.	Prioritized hierarchy of project threats categorized into Red (Critical), Amber (Moderate), and Green (Low) risk bands.
3. Quantitative Risk Modeling	Monte Carlo schedule simulation, Decision tree risk rollback, Sensitivity tornado diagrams.	Probabilistic confidence intervals for project completion date (e.g., P80 schedule and P90 cost contingency reserves).
4. Risk Response Planning	Four classic response postures: Avoid, Mitigate, Transfer (subcontracting/insurance), or Accept (contingency buffers).	Contingency Action Plans, Secondary Risk audits, and residual risk monitoring protocols.

13.2 Monte Carlo Schedule Simulation

While PERT relies on simplifying assumptions (such as focusing strictly on a single static critical path), complex real-world projects frequently feature multiple parallel pathways with nearly identical durations (near-critical paths). Under high variance, these secondary paths can easily surpass the theoretical critical path, causing severe schedule slippage—a phenomenon known as **Merge Event Bias**.

To overcome this limitation, enterprise project management utilizes **Monte Carlo Simulation**:

1. For each activity in the network, a probability distribution (e.g., Triangular, Beta, or Lognormal) is assigned based on empirical historical data.
2. A computer simulation executes thousands of randomized iterations (runs). In each iteration, a random duration is sampled for every activity according to its distribution, and complete forward/backward passes are computed.
3. The simulation records: (a) Total project duration frequency histogram; (b) **Criticality Index** of each activity (the percentage of total simulation runs in which that specific activity became critical).

Chapter 14: Contemporary Project Methodologies — Agile versus Predictive Waterfall

14.1 The Methodological Dichotomy

Operations researchers classify project delivery methodologies along a continuous spectrum ranging from predictive plan-driven frameworks to adaptive empirical frameworks:

Analytical Dimension	Predictive (Waterfall / Plan-Driven)	Adaptive (Agile / Value-Driven)
Scope Management	Strictly fixed upfront in comprehensive baseline contracts; alterations require formal Change Control Boards.	Dynamic, evolving Product Backlog continuously prioritized by customer business value.
Delivery Cadence	Single big-bang delivery at the final culmination of all scheduled activities.	Iterative, incremental delivery of Potentially Shippable Increments in short 2-to-4 week Sprints.
Customer Collaboration	Heavy formal contractual negotiation during initiation and client acceptance at final handover.	Continuous direct co-creation with an embedded Product Owner participating in daily stand-ups and sprint reviews.
Primary Tracking Metric	Variance against planned baselines (Earned Value SPI and CPI).	Team Velocity, Sprint Burn-down charts, and working software functionality delivered per sprint.
Best-Fit Problem Domain	Civil infrastructure construction, mechanical manufacturing, bridge building, defense contracts.	Software application development, digital marketing campaigns, artificial intelligence research.

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