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BBA | Semester 6 | UK6DSCMGT302

# Accounting for Managers

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# Module 2: Marginal Costing and Cost-Volume-Profit Analysis

Cost Behavior, CVP Analysis, Break-Even Calculations, and Managerial Decisions

## Chapter 1: Theoretical Foundations of Marginal Costing and Cost Behavior Dynamics

### 1.1 Definitional Architecture and Philosophical Evolution

In traditional financial accounting and absorption costing systems, all manufacturing costs—both variable direct operating expenditures and fixed indirect factory overheads—are absorbed into the unit cost of production. Consequently, fixed manufacturing overheads are capitalized and carried forward across balance sheet reporting periods inside unsold finished goods and work-in-progress inventory. This structural feature causes reported net accounting profits to fluctuate erratically based on production output volume rather than actual sales demand. For instance, an enterprise can report artificially inflated profits simply by overproducing inventory and storing it in warehouses, thereby deferring fixed factory overheads to future periods.

To eliminate this significant distortion and supply enterprise executives with rational, decision-relevant management intelligence, modern managerial accounting relies upon **Marginal Costing** (conventionally designated in American accounting literature as *Variable Costing* or *Direct Costing*). The Chartered Institute of Management Accountants (CIMA, London) formally defines marginal costing as "the ascertainment of marginal cost and of the effect on profit of changes in volume or type of output by differentiating between fixed costs and variable costs."

Under the marginal costing discipline, only variable costs (direct materials, direct labor, direct expenses, and variable factory overheads) are charged to products as inventoriable product costs. **Fixed Costs** are viewed fundamentally as **Period Costs**—incurred to maintain organizational capacity over chronological time—and are consequently expensed in full against the revenue of the accounting period in which they accrue. Finished goods and work-in-progress inventories are valued strictly at marginal manufacturing cost, preventing fixed cost deferral and ensuring that reported net profits reflect genuine commercial market sales performance.

### 1.2 Mathematical Classification and Segregation of Cost Behaviors

An uncompromising understanding of cost behavior within the relevant range of operating capacity is the primary prerequisite of marginal costing analysis. Costs are categorized into three distinct operational classifications:

- **Variable Costs:** Costs that fluctuate in direct, linear proportion to changes in production output volume. If factory throughput increases by 25%, total variable costs expand by exactly 25%; if plant production halts completely, total variable costs drop to zero. Crucially, *variable cost per unit remains constant* regardless of operational volume fluctuations within the defined relevant range of capacity.

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- **Fixed Costs:** Costs that remain invariant and constant in total monetary aggregate over a specific chronological timeframe regardless of fluctuations in production volume. Factory lease rentals, municipal property taxes, executive managerial salaries, and straight-line depreciation of machinery must be settled even if manufacturing operations are temporarily suspended. Consequently, *fixed cost per unit varies inversely with production output*: as production volume expands, fixed overhead is distributed across a greater number of physical units, causing unit fixed cost to decline asymptotically.
- **Semi-Variable (Mixed) Costs:** Complex operational expenditures that exhibit both fixed baseline and volume-responsive variable characteristics. Typical corporate examples include electricity utility tariffs (fixed meter standing charges plus per-unit kilowatt-hour consumption fees) and factory equipment maintenance (routine baseline servicing plus run-time operational wear and tear).

## 1.3 Mathematical Segregation Techniques for Mixed Costs

To implement marginal costing, management accountants must rigorously decompose semi-variable expenditures into their fixed and variable constituents. Three primary analytical techniques are deployed:

### 1. The High-Low Method

The high-low method compares historical operational activity at the highest and lowest recorded volume levels within the relevant range. The incremental cost difference is divided by the incremental volume difference to determine the unit variable cost:

$$\text{Variable Cost per Unit } (b) = \frac{\text{Total Cost at Highest Activity} - \text{Total Cost at Lowest Activity}}{\text{Highest Activity Units} - \text{Lowest Activity Units}}$$

$$\text{Fixed Cost Base } (a) = \text{Total Cost at Highest Activity} - [b \times \text{Highest Activity Units}]$$

**Operational Limitation:** The high-low method relies exclusively on two extreme data points, rendering it vulnerable to distortion by statistical outliers or unrepresentative operational shocks.

### 2. The Method of Least Squares (Linear Regression Analysis)

Linear regression provides statistical precision by incorporating all historical data observations across the relevant operational range. The cost function is expressed as $Y = a + bX$\$, where $Y$\$ represents total semi-variable cost, $X$\$ represents production volume, $a$\$ denotes the total fixed cost base, and $b$\$ represents the marginal variable cost coefficient per unit:

$$b = \frac{N \sum (XY) - (\sum X)(\sum Y)}{N \sum X^2 - (\sum X)^2}$$

$$a = \frac{\sum Y - b(\sum X)}{N}$$

**Where:** $N$ represents the number of historical monthly or quarterly observation periods.

### 3. Analytical Engineering and Inspection Method

Under this approach, industrial engineers and cost accountants conduct technical evaluations of manufacturing specifications, time-and-motion studies, and utility consumption schedules to establish physical input-output coefficients for variable components, isolating unavoidable baseline administrative overheads.

## 1.4 Foundational Assumptions and Strategic Advantages of Marginal Costing

The analytical validity of marginal costing rests upon five structural operational postulates:

1. All manufacturing, administrative, and distribution expenses can be segregated cleanly into fixed and variable elements.
2. Variable cost per unit remains perfectly constant at all operational volume levels, implying horizontal input price curves and linear variable cost behavior.
3. Total fixed costs remain completely static across the relevant range of operating capacity.
4. Selling price per unit remains constant regardless of sales quantity, assuming perfectly elastic product demand within the enterprise's market segment.
5. Either production output precisely matches sales volume, or inventory changes are evaluated strictly at variable manufacturing cost, ensuring no fixed costs are deferred.

The strategic benefits of marginal costing include: (a) eliminating misleading profit distortions caused by inventory level changes; (b) establishing clear visibility over **Contribution Margin** as the operational driver of enterprise profitability; (c) providing clean mathematical modeling for cost-volume-profit dynamics; and (d) delivering relevant differential cost data essential for strategic pricing, outsourcing, and special order decisions.

# Chapter 2: The Mathematical Architecture of Cost-Volume-Profit (CVP) Analysis

## 2.1 The Concept of Contribution and the Fundamental Profit Equation

The cornerstone metric of marginal costing is **Contribution Margin**. Contribution represents the surplus of sales revenue over total variable costs. It is termed contribution because it first contributes toward the recovery and liquidation of unavoidable fixed overheads; once fixed costs are fully liquidated, every additional rupee of contribution directly represents net operating profit.

> $$\text{Total Contribution } (C) = \text{Sales Revenue } (S) - \text{Total Variable Cost } (V)$$
>
> $$\text{Contribution per Unit } (c) = \text{Selling Price per Unit } (s) - \text{Variable Cost per Unit } (v)$$
>
> $$\text{Fundamental Profit Equation: Profit } (P) = \text{Contribution } (C) - \text{Fixed Cost } (F)$$
>
> $$S - V = F + P \implies s(Q) - v(Q) = F + P$$

> **Where:** $Q$ represents total physical unit volume sold.

## 2.2 The Profit-Volume (P/V) Ratio and Its Analytical Mechanics

The **Profit-Volume (P/V) Ratio** (or Contribution Margin Ratio) expresses the proportion of sales revenue available to cover fixed costs and generate operating profit. It represents one of the most critical diagnostic metrics in managerial economics:

$$\text{P/V Ratio} = \frac{\text{Contribution}}{\text{Sales}} \times 100 = \frac{S - V}{S} \times 100 = \frac{s - v}{s} \times 100$$

$$\text{Alternative Two-Period Formula: P/V Ratio} = \frac{\text{Change in Profit (or Contribution)}}{\text{Change in Sales Revenue}} \times 100$$

**Analytical Principle:** Because fixed costs remain constant between two operational periods within the relevant range, any change in total profit is entirely attributable to the change in contribution generated by sales volume shifts.

Strategic managerial methods for improving the P/V ratio include: (1) raising the unit selling price without precipitating a disproportionate drop in sales volume; (2) engineering operational efficiencies to reduce unit variable material or labor costs; (3) altering product sales mix toward higher-margin product lines; and (4) eliminating low-contribution product variants.

## 2.3 Mathematical Determination of the Break-Even Point (BEP)

The **Break-Even Point (BEP)** is the operational level of activity where total revenue exactly equals total operational costs, resulting in zero profit and zero loss ($P = 0$\$). At this critical operating volume, aggregate contribution margin exactly equals total fixed overheads.

$$\text{Break-Even Point in Units } (BEP_{units}) = \frac{\text{Total Fixed Costs } (F)}{\text{Contribution per Unit } (s - v)}$$

$$\text{Break-Even Point in Monetary Value } (BEP_{value}) = \frac{\text{Total Fixed Costs } (F)}{\text{P/V Ratio}} = BEP_{units} \times s$$

When enterprise management targets a specific predetermined monetary profit ($P_{target}$\$), the required sales volume is formulated as:

$$\text{Required Sales Volume (Units)} = \frac{F + P_{target}}{s - v}$$

$$\text{Required Sales Revenue (Rs.)} = \frac{F + P_{target}}{\text{P/V Ratio}}$$

If management specifies a target profit after corporate taxation (P_{after_tax\$), given a corporate income tax rate of $t$\$, the target pre-tax profit must be derived before applying the CVP equation:

$$\text{Required Sales Revenue (Rs.)} = \frac{F + \left(\frac{\text{Target Profit after tax}}{1-t}\right)}{\text{P/V Ratio}}$$

### 2.4 Margin of Safety and the Angle of Incidence

The **Margin of Safety (MOS)** represents the cushion by which actual or projected sales exceed the break-even sales volume. It indicates the magnitude of sales decline that an enterprise can absorb before incurring net operational losses:

$$\text{Margin of Safety in Value (Rs.)} = \text{Actual Sales Revenue} - \text{Break-Even Sales Revenue}$$

$$\text{Margin of Safety in Units} = \text{Actual Sales Units} - \text{Break-Even Sales Units}$$

$$\text{Margin of Safety Ratio} = \frac{\text{Actual Sales} - \text{Break-Even Sales}}{\text{Actual Sales}} \times 100 = \frac{\text{Net Profit}}{\text{P/V Ratio} \times \text{Actual Sales}}$$

$$\text{Direct Profit Relationship: Net Profit} = \text{Margin of Safety in Value} \times \text{P/V Ratio}$$

**Strategic Significance:** A high Margin of Safety indicates robust financial resilience, protecting the enterprise against sudden macroeconomic contractions. A low Margin of Safety warns management of extreme financial vulnerability to modest demand shocks.

In break-even graphical diagrams, the **Angle of Incidence** is the acute angle formed between the Total Sales Line and the Total Cost Line at the break-even intersection point. A wide angle of incidence, combined with an extensive margin of safety, reflects substantial profit-earning capacity per incremental rupee of sales beyond break-even. Conversely, a narrow angle of incidence signals sluggish profit generation and high variable cost friction.

## Chapter 3: Graphical Representation and Advanced Visual CVP Modeling

### 3.1 Construction Principles of the Conventional Break-Even Chart

The conventional break-even chart plots monetary values (Revenue and Costs in Rupees) on the vertical Y-axis and operational volume (Units of Output or Capacity Percentage) on the horizontal X-axis. The graphical construction follows four systematic steps:

1. **Horizontal Fixed Cost Line:** Drawn parallel to the horizontal X-axis at the level of total fixed overheads ($F$\$), illustrating that fixed costs do not vary with output volume.
2. **Total Cost Line:** Originates at the fixed cost intercept on the Y-axis and slopes upward to the right at an angle governed by the unit variable cost ($v$\$). The vertical distance between the Fixed Cost Line and Total Cost Line represents aggregate variable cost at any given output level.
3. **Total Revenue (Sales) Line:** Originates at the absolute origin $(0, 0)$\$ and slopes upward to the right at an angle determined by the unit selling price ($s$\$).

4. **Intersection and Break-Even Coordinates:** The point where the Total Revenue Line intersects the Total Cost Line represents the Break-Even Point. The area below this intersection represents the Operating Loss Zone, while the area above represents the Operating Profit Zone.

### 3.2 Alternative Graphical Formats: The Contribution Break-Even Chart

In the contribution break-even chart, the variable cost line is drawn first originating from the origin $(0, 0)$\$. The total cost line is then plotted parallel to the variable cost line, elevated by the vertical distance of total fixed overheads. This format visually highlights the total contribution area directly between the Total Revenue Line and the Variable Cost Line, providing immediate intuitive clarity on fixed cost absorption.

### 3.3 The Profit-Volume (P/V) Graph

The Profit-Volume graph simplifies CVP visualization by plotting net profit or net loss directly against sales volume. The horizontal axis represents sales volume at zero profit (the break-even line). At zero sales volume, the profit line originates at negative fixed cost ($-F$\$) on the vertical axis. The profit line crosses the horizontal axis at the break-even sales volume and climbs upward at a slope precisely equal to the P/V ratio.

## Chapter 4: Multi-Product CVP Analysis, Composite Break-Even, and Sales Mix Dynamics

### 4.1 Challenges of Multi-Product Operations

In modern industrial commerce, virtually all enterprises manufacture and market a diverse portfolio of multiple products, each possessing distinct unit selling prices, variable cost structures, and individual P/V ratios. Under multi-product conditions, a single break-even point in physical units cannot be computed directly unless an explicit, constant **Sales Mix Ratio** is maintained across all product lines.

### 4.2 Mathematical Determination of Composite P/V Ratio and Composite Break-Even

To analyze multi-product CVP relationships, management accountants calculate the **Composite (Weighted Average) P/V Ratio** based on the relative monetary sales contribution of each product line:

$$\text{Composite P/V Ratio} = \frac{\sum \text{Total Contribution of All Products}}{\sum \text{Total Sales Revenue of All Products}} \times 100$$

$$\text{Composite P/V Ratio} = \sum [(\text{Individual P/V Ratio of Product } i) \times (\text{Sales Weight of Product } i)]$$

$$\text{Overall Enterprise Break-Even Sales (Rs.)} = \frac{\text{Total Common Fixed Costs}}{\text{Composite P/V Ratio}}$$

**Where:** Sales Weight represents the fraction of total enterprise revenue generated by each respective product line.

When product proportions are defined in physical units rather than rupee values, the composite break-even point in units of the total product basket is calculated using the weighted average contribution margin per unit:

$$\bar{c} = \sum(c_i \times w_i)$$

$$\text{Composite BEP (Total Baskets)} = \frac{\text{Total Fixed Costs}}{\bar{c}}$$

**Where:** $c_i$ is unit contribution of product $i$\$, and $w_i$ is its relative unit mix proportion.

### 4.3 Impact of Sales Mix Shifts on Enterprise Profitability

A critical managerial insight of multi-product CVP analysis is that overall enterprise net profit and break-even revenue can alter dramatically even when aggregate sales revenue remains entirely unchanged. If the sales mix shifts toward higher-margin products (higher P/V ratios), the composite P/V ratio expands, lowering the enterprise break-even point and boosting net profits. Conversely, if sales shift toward low-margin commodity products, the composite P/V ratio deteriorates, increasing the break-even requirement and depressing net profitability.

## Chapter 5: Strategic Decision-Making Applications of Marginal Costing

### 5.1 Make or Buy Decisions (Outsourcing Evaluation)

Enterprise executives frequently evaluate whether to fabricate a component part internally within the factory or procure it from an external vendor. The decision hinges upon the comparative analysis of **Relevant Differential Marginal Costs** against the supplier's quotation:

- **Unutilized (Idle) Factory Capacity:** When plant facilities are underutilized and have no alternative profitable application, fixed factory overheads are unavoidable and sunk. Therefore, the component should be manufactured in-house if the *Marginal Cost of Manufacture (Direct Materials + Direct Labor + Direct Expenses + Variable Overheads) is less than the External Purchase Price*.
- **Fully Utilized Factory Capacity:** When internal machinery is operating at 100% capacity, manufacturing the component requires displacing existing profitable production. In this circumstance, the decision equation must explicitly incorporate the **Opportunity Cost (Forgone Contribution of Displaced Production)**:

  $$\text{Relevant Cost to Make} = \text{Variable Manufacturing Cost} + \text{Opportunity Cost of Displaced Production}$$

  **Decision Rule:** Procure externally if Supplier Purchase Price is lower than Relevant Cost to Make.

- **Qualitative Strategic Factors:** Management must also evaluate non-financial parameters, including supplier delivery reliability, component quality consistency, proprietary technology leak risks, and supplier pricing power after internal tooling is dismantled.

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## 5.2 Optimization of Product Mix under Limiting / Key Factors

A **Key Factor (or Principal Budget Factor / Limiting Factor)** is any operational resource that is restricted in supply and prevents the enterprise from expanding production indefinitely to satisfy total market demand. Key factors commonly include limited skilled machine hours, specialized direct labor hours, scarce imported raw materials, or restricted factory floor space.

When a limiting factor exists, maximizing total corporate net profit requires allocating the scarce resource to products that generate the highest **Contribution per Unit of the Key Factor**, rather than selecting products with the highest unit selling price or highest individual P/V ratio:

$$\text{Key Factor Profitability Index} = \frac{\text{Contribution per Unit of Product}}{\text{Consumption of Limiting Factor per Unit}}$$

**Decision Protocol:** Rank all competing product lines in descending order of Key Factor Profitability Index. Allocate available scarce resources sequentially to the highest-ranked product until its market demand is fully satisfied, proceeding down the ranking hierarchy until resources are exhausted.

## 5.3 Acceptance of Special Export Orders below Domestic Selling Price

During economic downturns or periods of excess factory capacity, enterprises frequently receive international export offers at prices significantly below prevailing domestic market rates. Traditional absorption costing managers often reject such business because the offered price falls below total absorption unit cost. Marginal costing establishes that *as long as the special export price exceeds incremental variable production cost* and does not cannibalize domestic price structures or trigger additional fixed commitments, the export order generates positive contribution and increases net corporate profit.

## 5.4 Discontinuation of an Unprofitable Department or Product Line

Financial statements prepared under absorption costing may show that a specific branch department or product line is operating at an accounting net loss after allocating corporate head-office overheads. Management must not make the hasty decision to terminate the division without marginal costing analysis. If the division generates a **Positive Contribution Margin ($S > V$)** that covers its own avoidable fixed costs and contributes toward common corporate overheads, terminating the department will actually *reduce total corporate net profit* by the amount of lost contribution.

## 5.5 The Plant Shutdown Decision

When an enterprise faces severe temporary economic collapse or supply chain disruptions, management must decide whether to continue operating at a temporary loss or suspend factory operations completely. Even if a factory is temporarily shut down, certain unavoidable fixed overheads continue to accrue (e.g., building security, insurance, property tax, maintenance of idle machinery). These are designated as **Unavoidable Fixed Costs**. The costs that can be eliminated by closing the plant are designated as **Discretionary (Avoidable) Fixed Costs**. The **Shut-Down Point** represents the minimum operational volume required to justify remaining open:

$$\text{Shut-Down Point (Units)} = \frac{\text{Total Fixed Costs} - \text{Unavoidable Shutdown Costs}}{\text{Contribution per Unit}} = \frac{\text{Avoidable Fixed Costs}}{s - v}$$

$$\text{Shut-Down Point (Sales Value)} = \frac{\text{Avoidable Fixed Costs}}{\text{P/V Ratio}}$$

**Decision Rule:** If anticipated market sales fall below the Shut-Down Point, the enterprise incurs lower total financial losses by temporarily closing the plant than by continuing operations.

## Chapter 6: Cash Break-Even Analysis and Operational Cost Indifference Point

### 6.1 Cash Break-Even Point

While the standard accounting break-even point establishes the sales volume required to avoid net accounting losses, the **Cash Break-Even Point** identifies the sales volume necessary to avoid cash deficits. In capital-intensive industries, substantial portions of fixed costs represent non-cash accounting charges (e.g., depreciation of plant and machinery, amortization of intangible assets, preliminary expense write-offs). The cash break-even point reflects the sales volume required to recover only the cash fixed overheads:

$$\text{Cash Fixed Costs} = \text{Total Fixed Costs} - \text{Non-Cash Charges (Depreciation and Amortization)}$$

$$\text{Cash Break-Even Point in Units} = \frac{\text{Cash Fixed Costs}}{\text{Cash Contribution per Unit}}$$

$$\text{Cash Break-Even Point in Value (Rs.)} = \frac{\text{Cash Fixed Costs}}{\text{Cash P/V Ratio}}$$

**Managerial Importance:** In times of severe credit tightness or liquidity crises, operating above the cash break-even point ensures enterprise survival, even if accounting net losses are recorded.

### 6.2 Operational Cost Indifference Point

When management evaluates two alternative production technologies or operational setups—for instance, Technology A utilizing labor-intensive methods with low fixed costs but high unit variable costs, versus Technology B utilizing automated robotic machinery with high fixed costs but low unit variable costs—the **Cost Indifference Point** identifies the specific operational volume where total costs under both alternatives are identical:

$$\text{Cost Indifference Point (Units)} = \frac{\text{Difference in Total Fixed Costs}}{\text{Difference in Unit Variable Costs}} = \frac{F_B - F_A}{v_A - v_B}$$

$$\text{Cost Indifference Point (Sales Value)} = \frac{\text{Difference in Total Fixed Costs}}{\text{Difference in P/V Ratios}}$$

**Decision Framework:** If anticipated production volume is *below* the indifference point, the lower-fixed-cost alternative (Technology A) is more economical. If anticipated volume is *above* the indifference point,

> the higher-fixed-cost automated alternative (Technology B) minimizes total unit production costs and maximizes operating leverage.

## Chapter 7: Comprehensive Worked Numerical Problems in Marginal Costing

### 7.1 Worked Problem 1: CVP, Break-Even, Target Profit, and Margin of Safety

Zenith Engineering Ltd. manufactures precision electronic controllers. The financial records provide the following baseline operational data for the current financial year:

- Unit Selling Price ($s$): Rs. 250.00
- Direct Raw Material per Unit: Rs. 75.00
- Direct Labor per Unit: Rs. 50.00
- Variable Manufacturing Overhead per Unit: Rs. 25.00
- Total Fixed Operating Overheads ($F$): Rs. 6,00,000 per annum
- Current Annual Output and Sales Volume: 8,000 units

**Required Calculations:**

1. Variable cost per unit and Contribution per unit.
2. Profit-Volume (P/V) Ratio.
3. Break-Even Point in units and in sales revenue.
4. Margin of Safety in units, value, and as a percentage of sales.
5. Required sales volume in units and value to earn a target pre-tax profit of Rs. 3,00,000.
6. Required sales revenue to earn a target post-tax profit of Rs. 2,80,000 assuming a corporate income tax rate of 30%.

**Step-by-Step Mathematical Resolution:**

*Step 1: Compute Unit Variable Cost and Unit Contribution:*

$$v = 75 + 50 + 25 = \text{Rs. } 150.00 \text{ per unit}$$

$$c = s - v = 250 - 150 = \text{Rs. } 100.00 \text{ per unit}$$

*Step 2: Calculate the Profit-Volume (P/V) Ratio:*

$$\text{P/V Ratio} = \frac{c}{s} \times 100 = \frac{100}{250} \times 100 = 40.0\%$$

*Step 3: Calculate the Break-Even Point (BEP):*

$$BEP_{units} = \frac{F}{c} = \frac{6,00,000}{100} = 6,000 \text{ units}$$

$$BEP_{value} = \frac{F}{\text{P/V Ratio}} = \frac{6,00,000}{0.40} = \text{Rs. } 15,00,000$$

*Step 4: Calculate the Margin of Safety (MOS):*

$$\text{Actual Sales Revenue} = 8,000 \times 250 = \text{Rs. } 20,00,000$$

$$\text{MOS (Units)} = 8,000 - 6,000 = 2,000 \text{ units}$$

$$\text{MOS (Value)} = 20,00,000 - 15,00,000 = \text{Rs. } 5,00,000$$

$$\text{MOS Percentage} = \frac{5,00,000}{20,00,000} \times 100 = 25.0\%$$

$$\text{Verification of Net Profit} = \text{MOS Value} \times \text{P/V Ratio} = 5,00,000 \times 0.40 = \text{Rs. } 2,00,000$$

*Step 5: Compute Required Sales for Pre-Tax Target Profit of Rs. 3,00,000:*

$$\text{Required Sales (Units)} = \frac{F + P_{target}}{c} = \frac{6,00,000 + 3,00,000}{100} = \frac{9,00,000}{100} = 9,000 \text{ units}$$

$$\text{Required Sales Revenue} = 9,000 \times 250 = \text{Rs. } 22,50,000$$

*Step 6: Compute Required Sales for Post-Tax Target Profit of Rs. 2,80,000 (Tax Rate 30%):*

$$\text{Target Pre-Tax Profit} = \frac{P_{after_tax}}{1-t} = \frac{2,80,000}{1-0.30} = \frac{2,80,000}{0.70} = \text{Rs. } 4,00,000$$

$$\text{Required Sales Revenue} = \frac{F + \text{Pre-Tax Profit}}{\text{P/V Ratio}} = \frac{6,00,000 + 4,00,000}{0.40} = \frac{10,00,000}{0.40} = \text{Rs. } 25,00,000$$

$$\text{Required Sales Units} = \frac{25,00,000}{250} = 10,000 \text{ units}$$

## 7.2 Worked Problem 2: Limiting Factor Optimization

Delta Manufacturing Corporation manufactures two industrial components, designated Alpha and Beta, using a specialized automated CNC milling machine. Total available machine capacity is strictly restricted to 4,800 machine hours for the upcoming operating quarter. Operational parameters are provided below:

| Operational Metric | Component Alpha | Component Beta |
|---|---|---|
| Maximum Market Demand | 2,000 units | 1,500 units |
| Selling Price per Unit | Rs. 300.00 | Rs. 450.00 |
| Direct Material per Unit | Rs. 100.00 | Rs. 180.00 |
| Direct Labor per Unit | Rs. 50.00 | Rs. 90.00 |
| Variable Overhead per Unit | Rs. 30.00 | Rs. 60.00 |
| Machine Hours Required per Unit | 2.0 hours | 3.0 hours |

**Required:** Determine the optimal production schedule that maximizes quarterly corporate contribution and profit under the machine-hour constraint.

**Step-by-Step Mathematical Resolution:**

*Step 1: Compute Contribution per Unit for Each Component:*

$$v_{Alpha} = 100 + 50 + 30 = \text{Rs. } 180.00 \implies c_{Alpha} = 300 - 180 = \text{Rs. } 120.00$$

$$v_{Beta} = 180 + 90 + 60 = \text{Rs. } 330.00 \implies c_{Beta} = 450 - 330 = \text{Rs. } 120.00$$

**Observation:** Both components yield an identical absolute contribution of Rs. 120.00 per unit.

*Step 2: Calculate Key Factor Profitability Index (Contribution per Machine Hour):*

$$\text{Index}_{Alpha} = \frac{c_{Alpha}}{\text{Machine Hours per Unit}} = \frac{120}{2.0} = \text{Rs. } 60.00 \text{ per machine hour}$$

$$\text{Index}_{Beta} = \frac{c_{Beta}}{\text{Machine Hours per Unit}} = \frac{120}{3.0} = \text{Rs. } 40.00 \text{ per machine hour}$$

**Ranking Priority:** Component Alpha generates Rs. 60.00 per scarce machine hour versus Rs. 40.00 for Component Beta. Therefore, Component Alpha is allocated **Rank 1** and Component Beta is allocated **Rank 2**.

*Step 3: Allocate Available Machine Hours (Total 4,800 Hours):*

- Satisfy full market demand for Rank 1 (Alpha): $2,000 \text{ units} \times 2.0 \text{ hours} = 4,000$ machine hours.
- Remaining Machine Hours for Beta: $4,800 - 4,000 = 800$ machine hours.
- Quantity of Beta Produced: $\frac{800 \text{ hours}}{3.0 \text{ hours/unit}} = 266.67 \implies 266$ completed units.

*Step 4: Total Contribution Generated under Optimal Production:*

$$\text{Contribution from Alpha} = 2,000 \times 120 = \text{Rs. } 2,40,000$$

$$\text{Contribution from Beta} = 266 \times 120 = \text{Rs. } 31,920$$

$$\text{Total Optimized Contribution} = 2,40,000 + 31,920 = \text{Rs. } 2,71,920$$

### 7.3 Worked Problem 3: Make or Buy Strategic Decision

Pioneer Appliances Ltd. manufactures commercial microwave ovens. The assembly requires an electronic thermostat unit, designated Sub-Assembly T-100. The current internal production cost per unit for 10,000 units per annum is structured as follows:

| Cost Element | Cost per Unit (Rs.) | Total Cost for 10,000 Units (Rs.) |
|---|---|---|
| Direct Material | 60.00 | 6,00,000 |
| Direct Labor | 40.00 | 4,00,000 |
| Variable Factory Overhead | 20.00 | 2,00,000 |
| Allocated Fixed Factory Overhead | 50.00 | 5,00,000 |
| **Total Unit Manufacturing Cost** | **170.00** | **17,00,000** |

An external specialized sensor supplier offers to supply 10,000 units of Sub-Assembly T-100 at a firm purchase price of Rs. 140.00 per unit.

**Decision Scenarios to Evaluate:**

1. **Scenario A:** The factory capacity released by outsourcing T-100 will remain idle, and fixed overheads cannot be reduced. Should Pioneer Appliances make or buy the component?
2. **Scenario B:** The released factory capacity can be leased out to an adjacent logistics firm for an annual rental income of Rs. 2,50,000. Should the component be made or bought?

**Step-by-Step Mathematical Resolution:**

*Resolution of Scenario A (Idle Capacity):*

$$\text{Relevant Marginal Cost to Make} = \text{Direct Material} + \text{Direct Labor} + \text{Variable Overhead}$$

$$\text{Relevant Cost to Make} = 60 + 40 + 20 = \text{Rs. } 120.00 \text{ per unit}$$

$$\text{External Supplier Purchase Price} = \text{Rs. } 140.00 \text{ per unit}$$

**Managerial Decision:** Because internal marginal cost (Rs. 120.00) is **Rs. 20.00 lower** than external purchase price (Rs. 140.00), Pioneer Appliances should **MAKE** the component internally. Outsourcing

under idle capacity would increase annual operating costs by $10,000 \times 20 = \text{Rs. } 2,00,000$. Allocated fixed overheads of Rs. 50.00 per unit are sunk and irrelevant.

*Resolution of Scenario B (Capacity Deployed for Rental Income):*

$$\text{Total Variable Cost to Make (10,000 units)} = 10,000 \times 120 = \text{Rs. } 12,00,000$$

$$\text{Add: Opportunity Cost (Forgone Rental Income)} = \text{Rs. } 2,50,000$$

$$\text{Total Relevant Cost to Make} = 12,00,000 + 2,50,000 = \text{Rs. } 14,50,000$$

$$\text{Effective Cost to Make per Unit} = \frac{14,50,000}{10,000} = \text{Rs. } 145.00 \text{ per unit}$$

$$\text{Total Cost to Buy Externally} = 10,00,000 \times 140 = \text{Rs. } 14,00,000$$

**Managerial Decision:** When opportunity cost is recognized, the effective cost to make becomes Rs. 145.00 per unit, which exceeds the supplier purchase price of Rs. 140.00. Pioneer Appliances should **BUY** the component externally and lease the idle factory space, generating a net annual economic gain of $14,50,000 - 14,00,000 = \text{Rs. } 50,000$.

## 7.4 Worked Problem 4: Operational Cost Indifference Point Analysis

Apex Manufacturing Corporation is planning to install a new fabrication line for an industrial fitting. Management is evaluating two competing manufacturing machinery options:

- **Technology A (Semi-Automated Machine):** Total Annual Fixed Cost ($F_A$): Rs. 4,00,000; Variable Cost per Unit ($v_A$): Rs. 60.00.
- **Technology B (Fully Automated Robotic System):** Total Annual Fixed Cost ($F_B$): Rs. 10,00,000; Variable Cost per Unit ($v_B$): Rs. 30.00.
- Both technologies produce an identical fitting that sells at Rs. 100.00 per unit.

**Required:**

1. Calculate the Cost Indifference Point in physical units and in sales value.
2. Determine which technology is optimal if annual projected sales volume is (a) 15,000 units, and (b) 25,000 units.
3. Calculate the Break-Even Point for each technology individually.

**Step-by-Step Mathematical Resolution:**

*Step 1: Compute the Cost Indifference Point:*

$$\text{Indifference Units} = \frac{F_B - F_A}{v_A - v_B} = \frac{10,00,000 - 4,00,000}{60 - 30} = \frac{6,00,000}{30} = 20,000 \text{ units}$$

$$\text{Indifference Sales Value} = 20,00,000 \times 100 = \text{Rs. } 20,00,000$$

> **Verification:** At 20,000 units, Total Cost A = $4,00,000 + (20,000 \times 60) = \text{Rs. } 16,00,000$. Total Cost B = $10,00,000 + (20,00,000 \times 30) = \text{Rs. } 16,00,000$. Costs are precisely identical.

*Step 2: Operational Evaluation at Differing Sales Volumes:*

- **At 15,000 Units (Below Indifference Point):**

> $$\text{Total Cost A} = 4,00,000 + (15,000 \times 60) = 4,00,000 + 9,00,000 = \text{Rs. } 13,00,000$$
>
> $$\text{Total Cost B} = 10,00,000 + (15,000 \times 30) = 10,00,000 + 4,50,000 = \text{Rs. } 14,50,000$$
>
> **Decision:** Technology A is superior, saving Rs. 1,50,000 in total costs.

- **At 25,000 Units (Above Indifference Point):**

> $$\text{Total Cost A} = 4,00,000 + (25,000 \times 60) = 4,00,000 + 15,00,000 = \text{Rs. } 19,00,000$$
>
> $$\text{Total Cost B} = 10,00,000 + (25,00,000 \times 30) = 10,00,000 + 7,50,000 = \text{Rs. } 17,50,000$$
>
> **Decision:** Technology B is superior, saving Rs. 1,50,000 in total costs due to superior operating leverage.

*Step 3: Individual Break-Even Analysis:*

> $$c_A = 100 - 60 = \text{Rs. } 40.00 \implies BEP_A = \frac{4,00,000}{40} = 10,000 \text{ units}$$
>
> $$c_B = 100 - 30 = \text{Rs. } 70.00 \implies BEP_B = \frac{10,00,000}{70} = 14,286 \text{ units}$$
>
> **Comparative Insight:** Technology A reaches break-even much earlier (10,000 units vs 14,286 units), demonstrating lower operating risk. However, once past 20,000 units, Technology B generates profit at a much faster rate ($c_B = \text{Rs. } 70$ vs $c_A = \text{Rs. } 40$).